METHODOLOGICAL FEATURES OF TEACHING RANDOM VARIABLES
AT THE COMPREHENSIVE SECONDARY SCHOOL***

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Abstract
In the evolving landscape of educational methodologies, the integration of complex mathematical concepts such as random variables into secondary school curriculums poses significant challenges and opportunities. As the global educational framework increasingly emphasizes the importance of a robust mathematical foundation, understanding and teaching stochastic elements within the school system has become crucial. This article delves into the nuanced methodological features essential for imparting knowledge of random variables effectively to secondary school students. We explore the historical context of mathematical education reforms in Armenia, highlighting the recent inclusion of stochastic topics in the school programs since 2004. The reformed educational standards introduced in 2020 and implemented in 2023 have further emphasized the need for a structured approach to teaching random variables, alongside other advanced mathematical topics. Through this lens, we examine the preparedness of educators, the adequacy of existing teaching methods, and the potential cognitive hurdles faced by students. Our analysis is rooted in both qualitative assessments and a theoretical framework that aims to bridge the gap between abstract mathematical theories and practical understanding, ensuring that students not only learn but also appreciate the significance of probability theory in everyday contexts.

Key words: comprehensive secondary school, probability theory, random variables, distributions, teaching methodology, educational methodologies, Mathematical education, stochastic elements, cognitive hurdles, qualitative

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INTRODUCTION

Taking into account the role and importance of stochastics in the modern scientific and educational system, it should be emphasized that currently ensuring the quality of stochastic education is a priority in the implementation of general education reforms. Speaking about the educational system of the Republic of Armenia, it should be noted that stochastic issues were not included in school programs until the last decades. Since 2004, when standards and subject programs of the "Mathematics" educational area were developed within the framework of the reforms of public education in Armenia (Mathematics, 2006, Elements of Algebra and Mathematical Analysis, Geometry, 2009), stochastic elements were included to some extent. However, research has shown that the introduction of the new content was a challenge for mathematics teachers because most of them were theoretically and methodologically unprepared to teach the new topics. Some teachers deliberately skipped sections related to probability theory and statistics (Vardazaryan, Minasyan, 2023).

Starting from 2020, the Ministry of Education, Science, Culture and Sport of the Republic of Armenia submitted for public discussion draft state subject standards and programs, which were implemented in general education schools in 2023 (Mathematics, 2020). The study of the package shows that in the standards and programs of the educational area "Mathematics" the content of probability-statistical material has been added and concretized.

In addition to the sections already familiar and understandable to the teachers, sections and topics that the teachers studied during their higher education were also included. One such topic is "Random Variables".

In particular, in the program of the school course of mathematics it is planned to teach the topics "Discrete random variable", "Discrete distributions", "Binomial distribution", "Mathematical expectation and dispersion of discrete random variables" in the 10th grade of secondary school in 2024. And in grade 11 it is planned to teach the topics "Continuous random variables", "Normal distribution", "Standardization", "Modeling with normal distribution" (Mathematics, 2023).

The question that needs to be clarified is whether students' prior knowledge will be sufficient to understand these concepts at this stage of learning. Can students at this stage of learning learn,
for example, the concept of countable sets? It should be noted that students of the humanities stream are unfamiliar with the concept of continuity, and the topic "Integrals" is envisaged by the program to be studied in grade 12 and only in streams with a physics and mathematics orientation (Vardazaryan, Minasyan, 2023).

In this regard, it is important to identify the methodological features of teaching the above topics and evaluate their role in terms of improving the effectiveness of learning.

**METHOD AND METHODOLOGY**

The methodological basis of the study was the concepts of applied direction of mathematics teaching, research aimed at improving the methodology of teaching mathematics, as well as theoretical issues of teaching.

**RESULTS AND DISCUSSION**

A random variable is a "fundamental stochastic concept" (Hernandez, Albert Huerta, & Batanero, 2006). Therefore, it is very important that students have a thorough understanding of the concept of a “Random variable”, which begins with a proper definition. The problems of studying the topic "Random Variables" have been comprehensively addressed by Lyutikas (1990), Adrian (2008), Brodsky (2008), Hernandez, Albert Huerta and Batanero (2006), Leviatan (2003), Batanero, Godino Juan and Roa (2004), Larsen. (2006) and others.

Teaching the topic "Random variables" is built on the content base of the secondary school course "Elements of algebra and mathematical analysis". This numerical function complements and expands students' understanding of functional dependencies.

In Armenia, starting from 2024-2025 academic year, discrete and continuous random variables and their numerical characteristics will be studied for the first time in the school mathematics course. The proposed new curriculum for 10th and 11th grades was tested in the schools of Tavush region and received positive feedback from teachers and students. But also some doubts were expressed related to the methodology of teaching these topics. We have already noted that teachers do not have sufficient methodological basis for teaching these topics. Therefore, it is necessary to organize professional development courses for teachers of 10th and 11th grades,
where appropriate methodological approaches for teaching these topics will be offered. These courses should be organized by competent organizations - higher education institutions.

Based on the European experience of teaching these topics in colleges, we propose some pedagogical approaches to solving the problem related to teaching discrete and continuous random variables. Since the students of 10th and 11th grades studied under the old subject program, there will naturally arise problems with the preliminary knowledge of students related to the knowledge of elements of probability theory and stochasticity. These problems will have to be solved in parallel with teaching new concepts, which will increase the workload for both students and teachers.

There are some common misconceptions about the concept of a Random Variable.

Students should be explained that the value of a random variable is a quantitative characteristic of an event. The following examples can be given for proper understanding of this fact.

1. The number of "crests" on a 3-fold coin flip is a random variable that can take the values 0, 1, 2, 3.
2. The number of cars passing on a highway in one hour is a random variable that can take the values 1, 2, 3, and other values.
3. The number of students participating in a math class is a random variable that can take one of the values 1, 2, 3,..., n (n is the number of students in the observed class).
4. A person's weight gain for a month is a random variable that can take values from a certain numerical interval.

Formation of the concept of a random variable should take place in stages: a variable that depends on the situation, a variable that refers to a random experiment, a numerical function that is determined on the basis of elementary results of the experiment (Minasyan, 2020).

**Definition.** A random variable is a variable whose values depend on the random outcome of some trial (Lyutikas, 1990):

Students are informed that random variables are denoted by capital Latin letters X, Y, Z,... and their probable values are denoted by lowercase Latin letters x, y, z,.....

It should be emphasized that the random variable X is a function in the elementary event space $\Omega$. 
Then it is appropriate to present the concepts of discrete and continuous random variables by examples.

**Definition.** A discrete random variable is a random variable that can take values of some finite or infinite numerical sequence (Lyutikas, 1990):

A discrete random variable \( X \) is said to be specified if all its possible values and the probabilities of accepting these values are listed. In this case, the distribution law of the random variable is said to be specified.

The law of distribution of a random variable is defined by the following formula:

\[
P(X = x_i) = p_i, \quad i = 1, 2, ..., n, ....
\]

Also, the distribution law of a discrete random variable can be presented in the form of a table (tab.1):

<table>
<thead>
<tr>
<th>( X )</th>
<th>( x_i )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>...</th>
<th>( x_n )</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( p_i )</td>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>...</td>
<td>( p_n )</td>
<td>...</td>
</tr>
</tbody>
</table>

For observation, the distribution law of a discrete random variable is also represented graphically, for which the points \((x_i, p_i)\) are represented in a rectangular Cartesian coordinate system and successively connected by segments. The resulting fraction is called the distribution polygon of the random variable \( X \).

For example. A random variable \( X \) is defined by the following distribution law (tab.2):

<table>
<thead>
<tr>
<th>( X )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0, 1</td>
<td>2</td>
<td>3</td>
<td>25</td>
<td>1</td>
<td>05</td>
</tr>
</tbody>
</table>

Draw the polygon distribution (fig.1).
In the process of teaching, students must be constantly reminded that as a result of an experiment, the random variable X necessarily takes any of the values $x_1, x_2, \ldots, x_n, \ldots$, hence,

$$p_1 + p_2 + \cdots + p_n + \cdots = 1.$$ 

Note also that the concepts "Random variable" and "Distribution of a random variable" are fundamental concepts for the study of Mathematical Statistics.

To reinforce the concept of "Law of distribution of a discrete random variable" we can offer the following task.

**Task.** During planting the distribution of apricot seedlings along the length of the row gave the following picture. Out of 10 plots of length 7 m on three of the average hit 2 seedlings, on four one seedling each, on one plot 3 seedlings, on the rest of the seedlings did not hit. Find the law of distribution of the number of apricot seedlings on 7 m long plots of the random variable X (Minasyan, 2020).

One of the important characteristics of random variables is the distribution function $F(x)$, which expresses the probability that a random variable X will take a value smaller than x, i.e. $F(x) = P(X < x)$. Geometrically, the distribution function is interpreted as the probability that a random point X is to the left of a given point x (Fig. 2).
The distribution function of any random variable has the following properties:

1. The function $F(x)$ is non-decreasing.
2. $F(-\infty) = P(X < -\infty) = P(\emptyset) = 0$.
3. $F(+\infty) = P(X < +\infty) = P(\Omega) = 1$.
4. The area of definition of the distribution function is the segment $[0, 1]$, because $0 \leq P(X < x) \leq 1$.
5. The probability that the random variable $X$ will fall on the segment $[\alpha, \beta]$ as a result of the experiment is calculated by the formula

$$P(\alpha \leq X < \beta) = F(\beta) - F(\alpha).$$

6. $\alpha < \beta, A = \{X < \alpha\}, B = \{X < \beta\}, C = \{\alpha \leq X < \beta\}$ (fig.3).

According to the figure $B = AUC$.

As the events $A$ and $C$ are incompatible, by the probability addition theorem

$$P(B) = P(A) + P(C)$$

or

$$P(X < \beta) = P(X < \alpha) + P(\alpha \leq X < \beta).$$

As a result, we get

$$P(\alpha \leq X < \beta) = P(X < \beta) - P(X < \alpha).$$

At the same time

$$P(X < \beta) = F(\beta) \text{ and } P(X < \alpha) = F(\alpha),$$

from which we obtain that

$$P(\alpha \leq X < \beta) = F(\beta) - F(\alpha).$$

So, if a discrete random variable takes values $x_1 < x_2 < \ldots < x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively, then its distribution function will take the form of
\[ F(x) = \begin{cases} 
0, & x < x_1 \\
p_1, & x_1 \leq x < x_2 \\
p_1 + p_2, & x_2 \leq x < x_3 \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
p_1 + p_2 + \cdots + p_{n-1}, & x_{n-1} \leq x < x_n \\
p_1 + p_2 + \cdots + p_n = 1, & x \geq x_n 
\end{cases} \]

It should be explained to students that in order to determine the distribution function of a discrete random variable, it is necessary to know its distribution law.

To reinforce this concept, the following task can be set.

*Task.* The number of customers who bought a new headache pill after watching an advertisement on TV is a random variable defined by the following table (tab. 3).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x_i )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>( p_i )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0, 2</td>
<td>0, 35</td>
<td>2</td>
<td>1</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find the distribution function of this quantity.

*Solution.* According to the formula, the distribution function has the following form:

\[ F(x) = \begin{cases} 
0, & x < 0 \\
0.1, & 0 \leq x < 10 \\
0.1 + 0.2, & 10 \leq x < 20 \\
0.1 + 0.2 + 0.35, & 20 \leq x < 30 \\
0.1 + 0.2 + 0.35 + 0.2, & 30 \leq x < 40 \\
0.1 + 0.2 + 0.35 + 0.2 + 0.1, & 40 \leq x < 50 \\
1, & x \geq 50 
\end{cases} \]

It should be noted that the graph of this function has a stepped form.

The above-mentioned approach to specifying the distribution function of a random variable allows studying discrete and continuous random variables with the same logic (Minasyan, 2020).

Then, addressing the most usual laws of distribution of a discrete random variable, their consolidation should be carried out by means of specially selected tasks. The first of the laws of distribution of a discrete random variable can be studied "Binomial distribution".

A probability distribution is called binomial if it is given by Bernoulli’s formula.
\[ P(X = m) = \binom{n}{m} p^m q^{n-m}, \]

where \( m \) is the number of experiments in which the random variable \( A \) occurred. For example, you might suggest considering the following task.

**Task.** As a result of quality inspection it was found out that on average 83 out of every 100 parts are not rejected. Draw the probability distribution law of the number of unrejected parts among 3 randomly selected parts.

**Continuous random variables.** Some misconceptions among students of probability theory have to do with continuous random variables, which are more difficult to understand than discrete random variables. One common misconception is to define a continuous random variable as a variable of an uncountable set (sometimes assumed to be an interval or a combination of intervals). This leads to the misconception that any random variable is either discrete or continuous. This misconception may also be related to a statistics course, where numerical data sets are categorized into two types: discrete and continuous.

Before introducing the concept of a continuous random variable, students can be given the following examples:

Human height is a quantity that takes different values from a certain interval depending on random circumstances.

The weight gain of a pet during a month is a random variable that can take values from a certain numerical interval.

**Definition.** A random variable that can take all values from a certain numerical interval is called a continuous random variable.

A more precise (mathematical) definition of a continuous random variable can be formulated as follows: if the distribution function \( F(x) \) is everywhere continuous and differentiable, then the random variable \( X \) is a continuous random variable (Gnedenko, 1969).

Unlike a discrete random variable, the values of a continuous random variable continuously fill the interval of a valid line. It is impossible to list all values of a continuous random variable and give their corresponding probabilities.

Note that the probability of each individual value of a continuous random variable \( X \) is zero. Indeed, take any point \( \alpha \) on the abscissa axis and the half-interval \([\alpha; \beta)\). We have

\[ P(\alpha \leq X < \beta) = F(\beta) - F(\alpha). \]
\( \beta \) is indefinitely approximated to the point \( \alpha \). We obtain

\[
P(X = \alpha) = \lim_{\beta \to \alpha} [F(\beta) - F(\alpha)].
\]

According to the property of continuous functions this limit is equal to zero, so

\[
P\{X = \alpha\} = 0.
\]

Probabilistic properties of a continuous random variable are given by a special function - the distribution density of the random variable. The definition of the density function is given by means of absolutely continuous random variable, but at the comprehensive school we can limit ourselves to the following definition.

**Definition:** The probability density \( p(x) \) is the derivative of the distribution function \( F(x) \) of a continuous random variable (Lyutikas, 1990):

Students should note that distribution and density functions are related by the formula \( p(x) = F'(x) \), so \( p(x) \) is sometimes called the differential distribution function and \( F(x) \) is sometimes called the distribution function or integral function.

Another common misconception among students is to think of a variable with a continuous distribution function as a continuous random variable. Note that the condition of continuity of the distribution function is a necessary but not a sufficient condition. Any singular random variable can be used as a counterexample, since the distribution function of such a variable is continuous and its derivative is almost always zero (Billingsley, 1995): Hence, such a random variable does not have a density function and is not continuous.

Turning to the numerical characteristics of random variables, it should be emphasized that in practice it is not always necessary to have a complete understanding of a random variable, it is enough to know only some of its characteristics that give a general idea of a random variable. The law of distribution fully characterizes a random variable. However, sometimes the law of distribution is unknown and we have to limit ourselves to a small amount of data. Sometimes it is even appropriate to use numbers that describe the random variable as a sum. Such numbers are called numerical characteristics of the random variable. Moreover, although quantitative data provide a set of possible numerical values of a random variable, they do not answer the question "which of the given values is most likely to be expected?" As a result, it becomes necessary to introduce the concept of "mathematical expectation".

The mathematical expectation serves as the center of the probability distribution of a random
variable. It is important to emphasize that the mathematical expectation is a point on the numerical axis around which the values of a discrete random variable are grouped.

It is also important to emphasize that mathematical expectation has a wider application when it is necessary to determine, for example, the average uptime of a device and other indicators.

It is also necessary to study the properties of mathematical expectation that facilitate the process of finding it:

1. \( M(C) = C \), где \( C = const \);
2. \( M(CX) = CM(X) \);
3. \( M(X \pm C) = M(x) \pm C \), where \( C = const \);
4. \( M(X \pm Y) = M(x) \pm M(Y) \).

Mathematical expectation allows us to judge only about the values of measurable objects in the case when the nature of the distribution of values may be different. In this connection, a new concept is introduced - the concept of dispersion, which estimates the scatter of values of random variables around their mathematical expectation. But before introducing this concept, we can consider the following examples: the same level of average annual precipitation in different geographical points, sufficiently distant from each other, or the same average score of students with low and high grades in different classes, and so on. Next, it is useful to consider dispersion properties, as they sometimes facilitate the calculation of variance (Minasyan, 2020):

Finding the mathematical expectation is not difficult. It is calculated using the following formula:

\[
M(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n + \cdots ,
\]

if \( X \) is a discrete random variable and

\[
M(X) = \int_{\alpha}^{\beta} xP(x)dx,
\]

if \( X \) is a continuous random variable defined in the interval \( (\alpha, \beta) \).

To reinforce this concept, the following example can be offered to 12th grade high school students:

*Find the mathematical expectation of a continuous random variable with density function* 

\[
p(x) = \frac{3}{125}x^2 \text{ given in the interval (0,5)}.
\]
Solution. \( M(X) = \int_{0}^{5} x \frac{3}{125} x^2 dx = \frac{3}{125} \int_{0}^{5} x^3 dx = \frac{3}{125} \cdot \frac{625}{4} = 3,75. \)

Solving the corresponding problems independently or with the help of the teacher allows students to easily master the skills of calculating mathematical expectation.

The mathematical expectation allows us to judge only the magnitudes of measurable objects, while the nature of the distribution of magnitudes may be different.

Example: Below are two discrete random variables \( X \) and \( Y \) with their distribution laws (tab. 4, tab. 5).

<table>
<thead>
<tr>
<th>( X )</th>
<th>-0,1</th>
<th>0,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0,5</td>
<td>0,5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( Y )</th>
<th>-25</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0,5</td>
<td>0,5</td>
</tr>
</tbody>
</table>

Although the mathematical expectations of \( X \) and \( Y \) are the same, \( MX = MY = 0 \), the possible values of \( X \) and \( Y \) are "scattered" differently around their mathematical expectations. The possible values of \( X \) are closer to their mathematical expectation than the values of \( Y \). In this connection, a new concept is introduced to estimate the scatter of values of a random variable around their mathematical expectation. This is the concept of variance.

The variance \( (DX) \) of a discrete random variable \( X \) is the mathematical expectation of the square of the variation of the random variable from its mathematical expectation.

\[ DX = M \left( (X - M(X))^2 \right) \]

or

\[ DX = (x_1 - M(X))^2 p_1 + (x_2 - M(X))^2 p_2 + \cdots + (x_n - M(X))^2 p_n + \cdots \]

In a basic course in probability theory, the variance of a continuous random variable is defined as follows:
The variance of a continuous random variable defined on the interval \((\alpha, \beta)\) is called the integral:

\[
DX = \int_{\alpha}^{\beta} (x - M(X))^2 p(x) \, dx \quad \text{(Gnedenko, 1969)}.
\]

**Example.** Find the variance of a continuous random variable \(X\) defined on the interval \((0,5)\) with density \(p(x) = \frac{3}{125} x^2\).

It is appropriate to consider dispersion properties because they sometimes simplify the calculation of dispersion.

1. \(D(C) = 0\), where \(C = \text{const}\),
2. \(D(CX) = C^2 DX\)
3. \(DX = M(X^2) - (M(X))^2\) (universal formula for calculating variance)
4. \(D(C+X) = DX:\)

So, the smaller the variance, the denser the values of specific experiments are clustered around the mathematical expectation, and this fact should be emphasized.

The introduction of the standard deviation should be explained by the fact that, compared to the dimensionality of the random variable, the variance is measured by a square unit, so when it is necessary to have a numerical characterization of the spread of possible values with the same dimensionality as the random variable itself, then the standard deviation is used. "Standard deviation" of a random variable \(X, \sigma\), is usually considered as the square root of its variance:

\[
\sigma = \sqrt{DX}.
\]

Students' attention should be paid to the fact that the standard deviation gives the range in which most of the values of a random variable are concentrated. Specially selected tasks contribute to the understanding of this fact. (Minasyan, 2020).

We can familiarize students with the law of normal distribution of a continuous random variable by means of concrete examples. The normal distribution in some sense occupies a central place in probability theory because, according to the central limit theorem, the sum of a sufficiently large number of relatively small random variables has a distribution close to the normal distribution. Its analytical representation allows us to establish a connection between algebraic and probabilistic components. The normal distribution has the following density:

\[
p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}.
\]
It should be noted that this law will further allow us to understand and make sense of statistical materials. The law of normal distribution can be interpreted by the following problem.

**Task.** On the basis of long-term clinical observations over monkeys, 100 blood tests were conducted to check the calcium content (%, mg) (calcium content is distributed according to the normal law). The lowest content was 13.5 and the highest was 14.3. The number of examinations in three groups was measured. The calcium content was 13.5-13.7 in the first, 13.8-14.0 in the second, and 14.1-14.3 in the third. Which group had the most number of examinations and why? (Minasyan, 2020).

Its geometric interpretation is of great importance for the formation and properties of the concept of the normal distribution density.

Most problems with normal distribution are reduced to determining the probability of a random variable falling into a given interval

\[ P(b \leq X \leq c) = \Phi \left( \frac{c-a}{\sigma} \right) - \Phi \left( \frac{b-a}{\sigma} \right). \]

**CONCLUSIONS**

Students who have acquired a basic knowledge of discrete and continuous variables at the comprehensive school can build on this knowledge, generalize and develop it to a higher level already in the university courses. Our experience shows that the described examples of random variables stimulate students' interest in the subject and develop their critical thinking.

Thus, within the framework of teaching elements of probability theory at the comprehensive school, the above methodological approaches to teaching the topic "Random variables" provide a deeper understanding of the basic probability concepts, realization of the practical significance of the system of tasks, increasing students' interest in them and, as a consequence, increasing the effectiveness of learning.

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